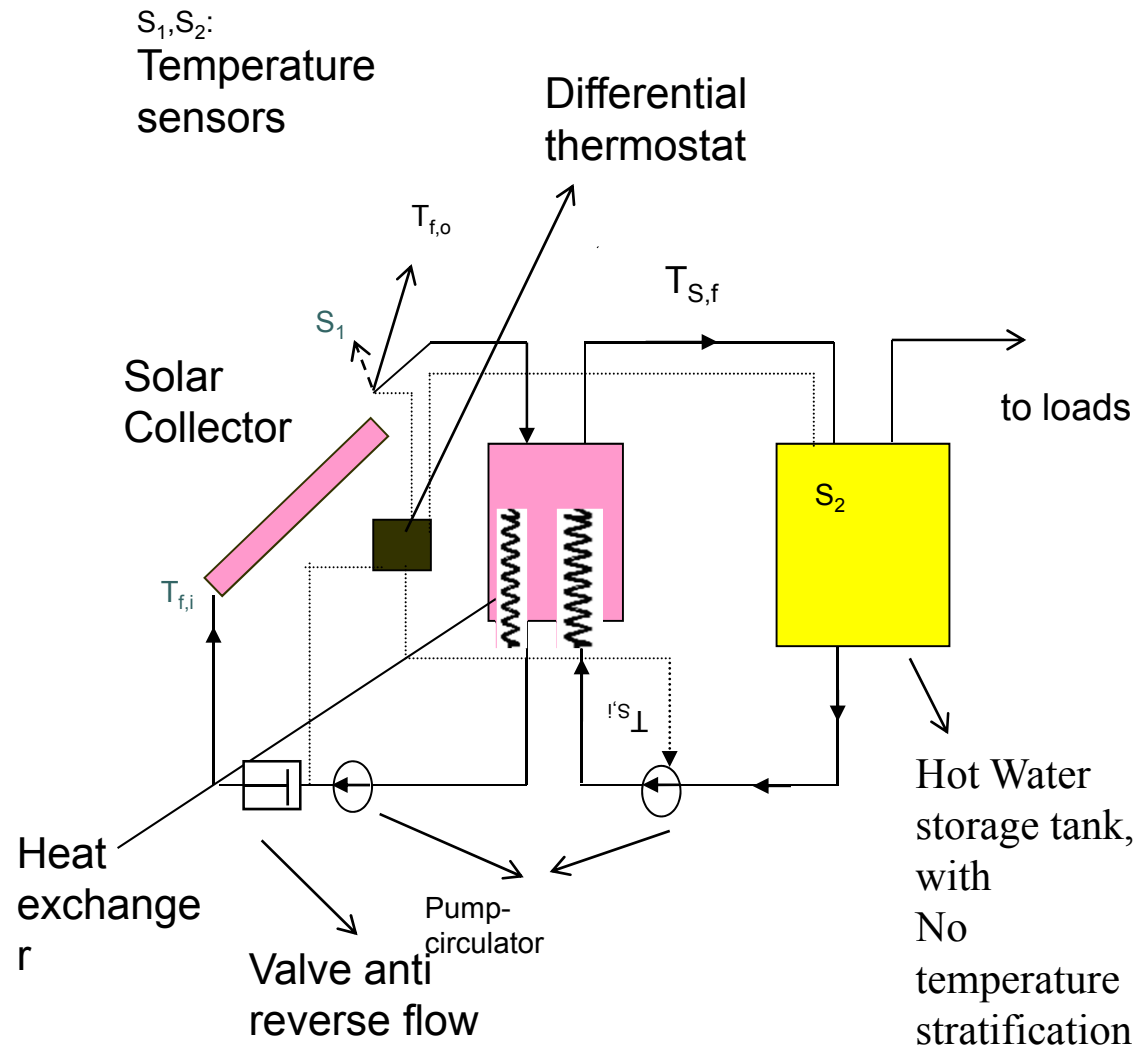


# Basics of thermal analysis of solar collector

## Simulation of operation of solar systems to determine the Thermal Gain.



**The useful thermal energy  $Q_u$  can be calculated from the relation:**

$$Q_u = (m C_p)_f (T_{f,o} - T_{f,i}) \quad (1.1)$$

**or normalized to solar collector surface, the thermal power output is given by:**

$$\dot{Q}_u = (\dot{m} C_p)_f (T_{f,o} - T_{f,i}) \quad (1.2)$$

**It can be also shown that the useful thermal gain from a solar collector is given by:**

$$\dot{Q}_u = F_R A_c [I_T (\tau\alpha) - U_L (T_{f,i} - T_a)] \quad (1.3)$$

**A similarity to the previous expressions holds for the hot water tank, too:**

$$\dot{Q}_u = (\dot{m} C_p)_s \cdot (T_{s,f} - T_{s,i}) \quad (1.4)$$

**or equivalently**

$$\dot{Q}_u = (\dot{m} C_p)_{\min} \cdot \varepsilon \cdot (T_{f,o} - T_{s,i}) \quad (1.5)$$

The **coefficient of effectiveness**,  $\varepsilon$ , is given by an expression in any Heat Transfer book,

$$\varepsilon = \frac{Q}{Q_{\max}} = \frac{(\dot{m}C_p)_c (T_{f,o} - T_{f,i})}{(\dot{m}C_p)_{\min} (T_{f,o} - T_{s,i})} = \frac{1 - e^{-NTU(1-C)}}{1 - C \cdot e^{-NTU(1-C)}} \quad (1.6)$$

From the expression (1.2) of calorimetry one gets:

$$C = (\dot{m}C_p)_{\min} / (\dot{m}C_p)_{\max} \quad NTU = (U_A \cdot A)_{\varepsilon v} / (\dot{m}C_p)_{\min} \quad (1.7)$$

$$T_{f,o} = T_{f,i} + \frac{\dot{Q}_u}{(\dot{m}C_p)_f}$$

**Substitute  $T_{f,i}$  from (1.7) to (1.3), then the equation which provides the Thermal Power stored in the system-tank takes the form:**

$$\dot{Q}_u = \frac{A_c F_R \left[ I_T (\tau \alpha) - U_L (T_{f,o} - T_a) \right]}{1 - \frac{A_c F_R U_L}{(\dot{m} C_p)_f}} \quad (1.8)$$

**We solve eq. (1.5) for  $T_{f,o}$  and we get:**

$$T_{f,o} = \frac{\dot{Q}_u}{(\dot{m} C_p)_{\min} \cdot \varepsilon} + T_{s,i} \quad (1.9)$$

**Basics of thermal analysis of solar  
collectors &  
Simulation of operation of solar  
systems to determine the Thermal  
Gain.**

**Prof. Socrates Kaplanis**

## References

Renewable Energy Systems : Theory and Intelligent Applications

Editors : S.Kaplanis, E.Kaplani

Nova Science Publishers, N.Y. , 2012

**Substitution of  $T_{f,o}$  to (.1.8), gives:**

$$\dot{Q}_u = \frac{A_c F_R \left[ I_T(\tau\alpha) - U_L \cdot \left[ \frac{\dot{Q}_u}{(\dot{m} C_p)_{\min} \cdot \epsilon} + T_{s,i} - T_a \right] \right]}{1 - \frac{A_c \cdot F_R U_L}{(\dot{m} C_p)_f}} \quad (1.10)$$

**The expression (1.10) can be easily simplified to:**

$$\dot{Q}_u = \frac{A_c F_R [I_T(\tau\alpha) - U_L (T_{s,i} - T_a)]}{1 + \frac{A_c F_R U_L}{(\dot{m} C_p)_f} \times \left[ \frac{(\dot{m} C_p)_f}{(\dot{m} C_p)_{\min} \cdot \epsilon} - 1 \right]} \quad (1.11)$$



**We define a new parameter,  $F'_R$ :**

$$F'_R = \frac{F_R}{1 + \frac{A_c F_R U_L}{(\dot{m} C_p)_f} \times \left[ \frac{(\dot{m} C_p)_f}{(\dot{m} C_p)_{\min} \cdot \varepsilon} - 1 \right]} \quad (1.12)$$

**Hence, the expression (1.11) is simplified as to:**

$$\dot{Q}_u = A_c * F_R \left[ I_T(\tau\theta) - U_L(T_{sj} - T_u) \right] = F_R A_c * \frac{F'_R}{F_R} \left[ I_T(\tau\theta) - U_L(T_{sj} - T_u) \right] \quad (1.13)$$

**Let us consider a small time period the solar collector system operates. Then, the mean water temperature in the storage tank is determined by:**

$$T_s = \frac{T_{s,i} + T_{s,f}}{2} \quad (1.14)$$

**The heat delivered by a collector,  $A_c$ , in a period,  $\Delta\tau$ , to the tank may be determined by:**

$$Q_{u,\Delta\tau} = (MC_p)_s (T_{s,f} - T_{s,i}) = \int_{\tau}^{\tau+\Delta\tau} \dot{Q}_u dt \quad (1.15)$$

**We divide both sides of (.1.15) over  $A_c$  to normalize the expression. Then**

$$\mathbf{q}_{u,\Delta\tau} = (\mathbf{m C}_p)_s (\mathbf{T}_{s,f} - \mathbf{T}_{s,i}) \quad (1.16)$$

**or equivalently**

$$\mathbf{T}_{s,f} - \mathbf{T}_{s,i} = \mathbf{q}_{u,\Delta\tau} / (\mathbf{m C}_p)_s \quad (1.17)$$

Substitute  $T_{s,f}$  from (1.17) to (1.14). We get:

$$T_s = T_{s,i} + \frac{Q_{u, \Delta\tau}}{(2 m C_P)_s} \quad (1.18)$$

$T_s$  is the mean temperature of the water in the tank in the above time interval. Integration of (1.13 or 1.15) for this time period gives:

$$Q_{u, \Delta\tau} = F_R' \times A_c [H_n(\tau\alpha) - U_L (T_s - \bar{T}_\alpha) \Delta\tau] \quad (1.19)$$

Substitute  $T_s$  from (1.18) to (1.19). We get:

$$q_{u,\Delta\tau} = \frac{F_R' [H_n(\overline{\tau\alpha}) - U_L (T_{s,i} - \overline{T_a}) \Delta\tau]}{1 + \frac{F_R' U_L}{2(m C_p)_s} \times \Delta\tau} \quad (1.20)$$

# Let us analyze a real case

A Solar Collector System, as the one shown in the 1<sup>st</sup> figure, has parameters:  $F_R U_L = 3.5 \text{ W/m}^2 \cdot \text{K}$  and  $F_R (\tau\alpha)_n = 0.69$

and is placed at horizontal position in Pyrgos.

The storage tank has capacity  $50 \text{ l/m}^2$ .

Let the storage tank temperature at 7:30 be  $20 \text{ C}$ .

Please determine the hourly temperature in the tank and the hourly efficiency.

Data input: ambient temperature,  $T_a$ , and the mean hourly global solar radiation,  $H_n$ . Values are given in the Table below

Data input and values of basic quantities as provided by the iteration procedure to be outlined below.

Time	$T_{\alpha}$ °C	$H_n$ ( $\frac{\text{kJ}}{\text{m}^2}$ )	$q_u$ ( $\frac{\text{kJ}}{\text{m}^2}$ )	$T_{s,i}$ °C	$T_{s,f}$ °C	$N=q_u/H_n$
<b>18.4.1999</b>						
<b>7:30 – 8:30</b>	<b>15.0</b>	<b>720</b>	<b>421</b>	<b>20</b>	<b>22</b>	<b>0.58</b>
<b>8:30 – 9:30</b>	<b>15.5</b>	<b>1476</b>	<b>909</b>	<b>22</b>	<b>26</b>	<b>0.61</b>
<b>9:30 – 10:30</b>	<b>16.5</b>	<b>1980</b>	<b>1206</b>	<b>26</b>	<b>32</b>	<b>0.60</b>
<b>10:30 – 11:30</b>	<b>17.0</b>	<b>2484</b>	<b>1479</b>	<b>32</b>	<b>39</b>	<b>0.59</b>
<b>11:30 – 12:30</b>	<b>17.5</b>	<b>2844</b>	<b>1640</b>	<b>39</b>	<b>46</b>	<b>0.57</b>
<b>12:30 – 13:30</b>	<b>18.0</b>	<b>3240</b>	<b>1816</b>	<b>46</b>	<b>55</b>	<b>0.56</b>
<b>13:30 – 14:30</b>	<b>19.0</b>	<b>3250</b>	<b>1729</b>	<b>55</b>	<b>63</b>	<b>0.53</b>
<b>14:30 – 15:30</b>	<b>19.0</b>	<b>2968</b>	<b>1439</b>	<b>63</b>	<b>70</b>	<b>0.48</b>
<b>15:30 – 16:30</b>	<b>18.0</b>	<b>2412</b>	<b>971</b>	<b>70</b>	<b>75</b>	<b>0.40</b>
<b>16:30 – 17:30</b>	<b>17.5</b>	<b>1800</b>	<b>498</b>	<b>75</b>	<b>77</b>	<b>0.27</b>
<b>17:30 – 18:30</b>	<b>17.0</b>	<b>1210</b>	<b>68</b>	<b>77</b>	<b>78</b>	<b>0.05</b>

To determine the quantities  $T_{s,f}$ ,  $T_{s,i}$

A. Time Interval 7:30 – 8:30 am

**Step 1st** : We determine the normalized useful heat by  
(1.20).

$$q_{u,\Delta\tau} = \frac{720 \text{kJ/m}^2 \times 0.69 - 3.5 \frac{\text{W}}{\text{m}^2 \text{K}} \times (20^0 - 15^0) \times 3600 \text{s}}{\left[ 1 + \frac{3.5 \frac{\text{W}}{\text{m}^2 \text{K}} \times 3600 \text{s}}{2 \times 50 \frac{\text{kg}}{\text{m}^2} \times 4180 \frac{\text{J}}{\text{kgK}}} \right]} = 421 \text{kJ/m}^2 \quad (1.21)$$



**Step 2nd : We determine tank temperature,  $T_{s,f}$ , at the end of the 1<sup>st</sup> interval 8:30 amusing expression (1.17)**

$$T_{s,f} = 20^{\circ}\text{C} + \frac{421,000\text{J/m}^2}{50\text{kg/m}^2 \times 4180\text{J/kgm}^2} = \mathbf{22^{\circ}\text{C}} \quad (1.22)$$

**Step 3rd : Determine efficiency,  $\eta$ , during this short period by :**

$$\eta = \frac{Q_{u,\Delta\tau}}{A_c H_n} = \frac{q_{u,\Delta\tau}}{H_n} \quad \eta = \frac{q_{u,\Delta\tau}}{H_n} = \frac{421\text{kJ/m}^2}{720\text{kJ/m}^2} = \mathbf{0.58} \quad (1.23)$$

## B. Time Interval 8:30 – 9:30am

We follow the same procedure as before. We put for this period 8:30-9:30 as  $T_{s,i}$ , the  $T_{s,f}$  value of the previous interval.

Determine  $q_{u, \Delta\tau}$  from (1.20)

$$q_{u, \Delta\tau} = \frac{1476 \text{kJ} / \text{m}^2 \times 0.69 - 3.5 \frac{\text{W}}{\text{m}^2 \text{K}} \times (22^0 - 15.5^0) \times 3600 \text{s}}{1.03} = 909 \text{kJ} / \text{m}^2$$

(1.24)

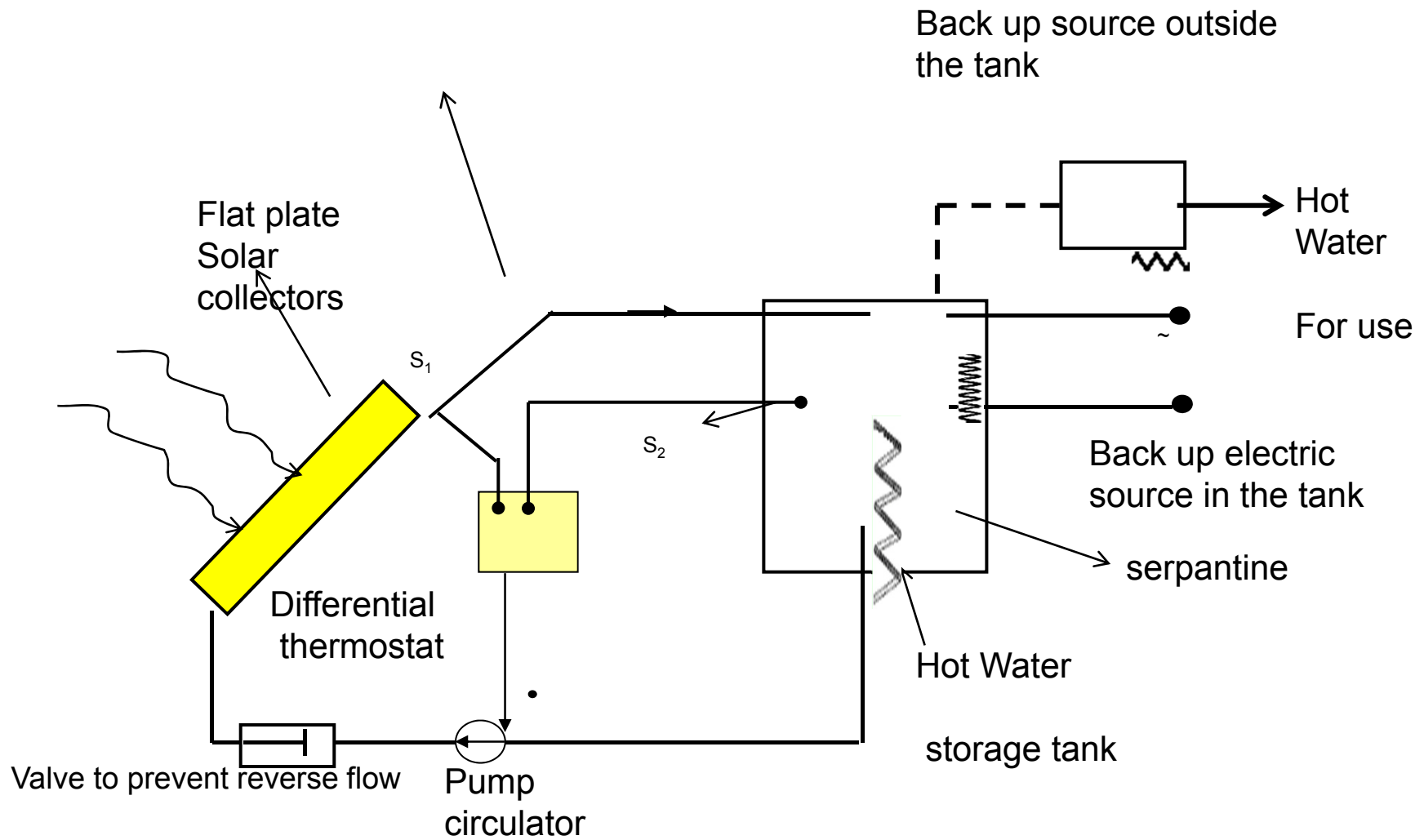
**Determine  $T_{s,f}$  from (1.17)**

$$T_{s,f} = 22^{\circ}\text{C} + \frac{909000 \frac{\text{J}}{\text{m}^2}}{50 \frac{\text{kg}}{\text{m}^2} \times 4180 \frac{\text{J}}{\text{kg}^{\circ}\text{C}}} = 26^{\circ}\text{C} \quad (1.25)$$

**Then, the efficiency is estimated by:**

$$\eta = \frac{q_{u, \Delta\tau}}{H_n} = \frac{909 \text{ kJ} / \text{m}^2}{1476 \text{ kJ} / \text{m}^2} = 0.61 \quad (1.26)$$

$s_1, s_2$ : temperature sensors



## 2. A generalized analysis to consider the Load, too.

Let us consider  $q_s$  as the net stored heat normalized to collector surface; that is when the Load,  $Q_L$ , is subtracted. Correspondingly, the thermal load per collector surface is denoted by, ( $q_L = Q_L / A_c$ ). Then it holds :

$$q_s = q_{u, \Delta\tau} - q_L \quad q_{u, \Delta\tau} = q_s + q_L \quad (2.1)$$

Following the procedure as for expression (1.18) there is given that:

$$T_s = T_{s, i} + \frac{q_s}{2(m C_P)_s} \quad (2.2)$$

2. A generalized analysis to consider the Load, too.

**Substitute Ts to (1.19). Then, the expression (1.20) is modified and due to (2.1) we get :**

$$q_s = \frac{F'_R [H_n(\overline{\tau\alpha}) - U_L (T_{s,i} - \overline{T\alpha}) \Delta\tau] - q_L}{1 + \frac{F'_R U_L}{2(m C_p)_s} \Delta\tau} \quad (2.3)$$

**We substitute (2.3) in (2.1) and we finally get the generalized iterative formula:**

$$q_{u,\Delta\tau} = \frac{F'_R [H_n(\overline{\tau\alpha}) - U_L (T_{s,i} - \overline{T\alpha}) \Delta\tau]}{1 + \frac{F'_R U_L}{2(m C_p)_s} \Delta\tau} + \frac{q_L}{1 + \frac{2(m C_p)_s}{F'_R U_L \Delta\tau}} \quad (2.4)$$



# References

Renewable Energy Systems: Theory and Intelligent Applications  
Editors S. Kaplanis, E. Kaplani  
Nova Science Publishers, N.Y., 2012